# Teaching mathematically gifted students in secondary schools: A systematic literature review

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## Motivation

School performance studies such as PISA show that the proportion of pupils with very good mathematical abilities has decreased significantly in Germany over the last 10 years (Diedrich et al., 2023). This may be related to inadequate or inappropriate support for mathematically gifted pupils.

While there are many suggestions for mathematical support programs in the literature, often based on practical experience, the empirical evidence regarding relevant design features of support programs still seems to be limited (Leikin, 2021), especially at secondary level.

## Research Questions

We would like to address the following questions:

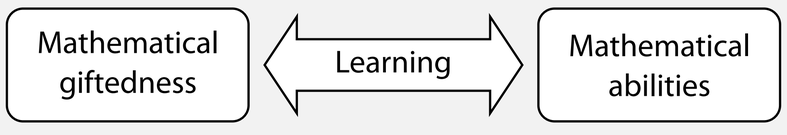
1. Which design features of learning environments for mathematically gifted students in secondary schools can be identified in empirical research literature? Which of them are effective?
2. To what extent do these design features correspond to the five principles for high-quality mathematics teaching according to Prediger et al. (2022)?

## Theoretical background

### Mathematical Giftedness

*Mathematical giftedness* is the individual potential to develop mathematical abilities.

*Learning* is a core process in the realization of mathematical giftedness. Therefore, stimulating learning processes is essential for realizing mathematical potential.

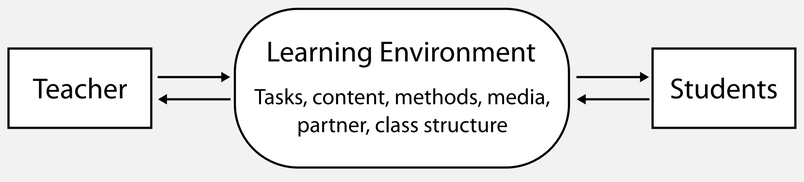



(Ulm & Zehnder, 2020)

### Learning Environment

Teachers design *learning environments* to stimulate learning processes. They define tasks, content or methods for the teaching and learning process.

Providing students with a learning environment is one way to stimulate learning processes.




(Ulm & Zehnder, 2020)

### Principles for high-quality mathematics teaching

**Conceptual Focus**
Develop conceptual understanding

**Cognitive Demand**
Engage in higher-order thinking processes

**Student Focus and Adaptivity**
Consider individual learning stages

**Longitudinal Coherence**
Organize long-term learning trajectories

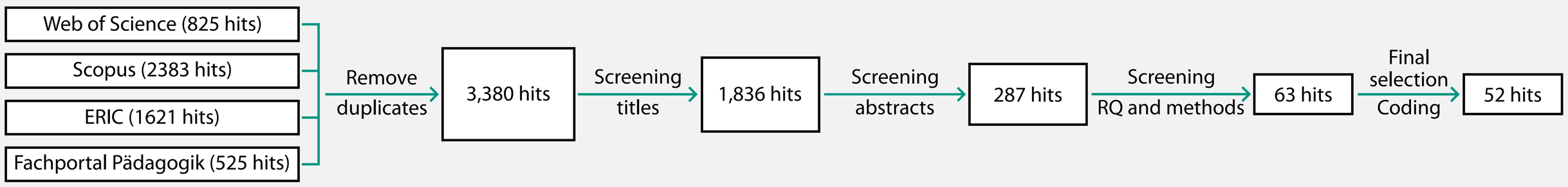
**Enhanced Communication**
Engage in mathematical communication

(Prediger et al., 2022)

## Methods

The basis for answering the research questions is a systematic literature review in four databases (Newman & Gough, 2020). The following search string, as given as an example for Web of Science, was the basis of the search:

(mathemati*) **AND** (gift* OR promis* OR begab* OR talent*) **AND** (teach* OR educat* OR nurtur* OR program* OR förder* OR unterricht*) **AND** (pupils OR students OR schüler*)




## Results

### Year of publication

| | | |
|---|---|---|
| 2021 – 2024 | 10 | (19 %) |
| 2011 – 2020 | 18 | (35 %) |
| 2001 – 2010 | 7 | (13 %) |
| 1991 – 2000 | 10 | (19 %) |
| 1981 – 1990 | 6 | (12 %) |
| 1971 – 1980 | 1 | (2 %) |

### Grades studied

The hits are almost evenly distributed in terms of grades.

> Students appreciated "the focus on having them understand *why* certain mathematical approaches or algorithms are used as opposed to merely *how*"
>
> (Tretter, 2003, p. 32)

> "opportunities to use their own strengths and experiences when working on rich tasks are of great importance for gifted students"
>
> (Simensen & Olsen, 2024, p. 13)

> "challenge … and high-level thinking should be integrated to the problem-solving task"
>
> (Ozdemir & Isiksal Bostan, 2021, p. 140)

> "Thus, there is a need for attention to task selection, opportunities for solution reconciliation … for effective community participation"
>
> (Diezmann & Watters, 2001, p. 27)

## Outlook

### Principles for mathematics teaching

- Do the five principles have different relevance when designing learning environments for mathematically gifted students?
- Are there further principles unique for gifted education?

### Design of learning environments

- Is there a focus on specific components in the design of a learning environment for mathematically gifted students?
- What specific design recommendations can be derived from the findings?

## Literature

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